Exercise 1: Different Approaches to Decision Making

Under what circumstances would you use any of the following statistical approaches to decision making?

1. frequentist: identify Wald's admissible decisions
2. Bayesian: maximise posterior expected utility
3. robust Bayesian: maximise posterior expected utility but also check the sensitivity of your conclusion against changes in the prior

Discuss your answer with your neighbour.

Exercise 2: Verify Wald's Theorem in the Boat Example

Consider again the boat example that we discussed.

utility function

$U(d,x)$	$x = 0.5$	$x = 2$
$d =$ boat	3	-1
$d =$ no boat	0	0

likelihood

$p(y \mid x)$	$y = 0.5$	$y = 2$
$x = 0.5$	0.9	0.1
$x = 2$	0.3	0.7

We established that the following strategies were optimal Wald strategies: We established that $\delta(y = 0.5) =$ boat, $\delta(y = 2) =$ no boat was an optimal Bayes strategy under prior $p(x = 0.5) = 0.4$, $p(x = 2) = 0.6$.

Verify that you get all Wald's admissible decisions by varying the prior.

Hint: repeat the analysis from the lecture for the following two priors:

$p(x)$	$x = 0.5$	$x = 2$
	1	0

$p(x)$	$x = 0.5$	$x = 2$
	0	1

Exercise 3: Machinery, Overtime, or Nothing?

Consider again the same very simple example. We have done additional market research, and we now know that demand will increase with probability at least 0.6, and at most 0.65.

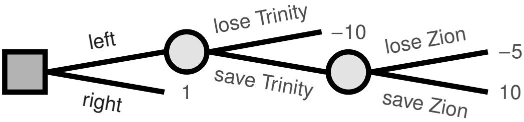
What advice can we give the manager now? Investigate with each optimality criterion.

Hint: $\mathcal{M} =$

	p_1	p_2
increase	0.6	0.65
stay	0.4	0.35

Exercise 4: Saving Zion (Or Maybe Not?)

There are two doors. The door to your right leads to the Source and the salvation of Zion. The door to your left leads back to the Matrix, to her…and to the end of your species. As you adequately put, the problem is choice. But we already know what you are going to do, don't we?



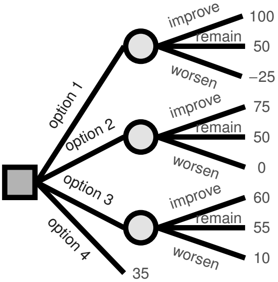
$\mathcal{M} =$

	p_1	p_2	p_3
lose Trin	0.1	0.4	0.3
save Trin & lose Zion	0.45	0.3	0.2
save Trin & save Zion	0.45	0.3	0.5

Left, or right? Investigate with your favorite optimality criterion.

Exercise 5: A Risky Investment

You have the option to invest some money. The market can either improve, remain, or worsen. The set of probabilities for your lower prevision are tabulated below. You have the choice between 4 options, summarized in the decision tree below.



$\mathcal{M} =$

	p_1	p_2
improve	0.0	0.3
remain	0.6	0.3
worsen	0.4	0.4

Which options should you definitely not consider? First consider interval maximality, then consider robust Bayes maximality. Which of these two criteria gives the better answer?

Exercise 6 (*)

Let X be any gamble, with lower prevision L and upper prevision U. Let c be any constant. Suppose you have the choice between the uncertain gain X, or the certain gain c.

Under each of the criteria, determine which of X or c (or both!) are optimal, under the following circumstances:

- $c < L$
- $L < c < U$
- $c > U$

In Exercise 5, option 4 corresponds to an investment without risk, as it yields the value $c = 35$ independently of the market, however, we found that this value was too low relative to the other options to be optimal.

For what values for c would you change your mind? Again, investigate this using each of the criteria, for $c < 37$, $37 < c < 37.5$, $37.5 < c < 38.5$, *and* $38.5 < c$.

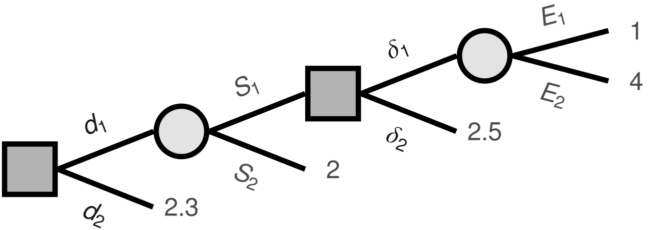
Exercise 7

State one advantage, and one disadvantage, of solving a sequential decision problem by normal form backward induction, compared to solving it by normal form.

Can you think of a situation in which normal form backward induction would be less efficient than normal form?

Exercise 8

Solve the following sequential decision problem for robust Bayes maximality, using either normal form, or normal form backward induction.



$\mathcal{M} =$

	p_1	p_2
$S_1 E_1$	0.2	0.1
$S_1 E_2$	0.3	0.4
S_2	0.5	0.5

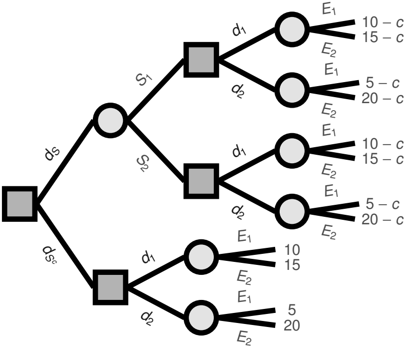
[Hint: $\mathcal{M}|S_1 =$

	p_1	p_2
E_1	0.4	0.2
E_2	0.6	0.8

]

Exercise 9 (*)

Tomorrow, a subject is going for a walk in the lake district. It may rain (E_1), or not (E_2). The subject can either take a waterproof (d_1), or not (d_2). But the subject may also choose to buy today's newspaper, at cost c, to learn about tomorrow's weather forecast (d_S), or not (d_{S^c}), before leaving for the lake district. The forecast has two possible outcomes: predicting rain (S_1), or not (S_2). *Solve for robust Bayes maximality, with* $c = 1$.



$\mathcal{M} =$

	p_1	p_2	p_3	p_4
$S_1 E_1$	0.378	0.378	0.378	0.478
$S_1 E_2$	0.162	0.162	0.262	0.162
$S_2 E_1$	0.072	0.172	0.072	0.072
$S_2 E_2$	0.388	0.288	0.288	0.288

Exercise 10 (**)

Consider again the lake district exercise.

For which values of c is it no longer robust Bayes maximal to buy the newspaper?

(This is the value of information of the newspaper.)