Decision Making Under Severe Uncertainty: Solutions to Exercises

A Very Simple Example

Example from lecture:

	increase	*stay*	*p0*	*p1*	*lpr*	*upr*
increase			0.5	0.8		
stay			0.5	0.2		
machinery	440	260	350	404	350	404
overtime	420	300	360	396	360	396
nothing	370	370	370	370	370	370

- Gamma-maximin: {nothing}
- Gamma-maximax: {machinery}
- interval maximal: {machinery, overtime, nothing}
- robust Bayes maximal: {machinery, overtime, nothing}

After market research:

	increase	*stay*	*p0*	*p1*	*lpr*	*upr*
increase			0.6	0.65		
stay			0.4	0.35		
machinery	440	260	368	377	368	377
overtime	420	300	372	378	372	378
nothing	370	370	370	370	370	370

- Gamma-maximin: {overtime}
- Gamma-maximax: {overtime}
- interval maximal: {machinery, overtime}
- robust Bayes maximal: {overtime}

Saving Zion

	A	*B*	*C*	*p0*	*p1*	*p2*	*lpr*	*upr*
A				0.1	0.4	0.3		
B				0.45	0.3	0.2		
C				0.45	0.3	0.5		
left	-10	-5	10	1.25	-2.5	1	-2.5	1.25
right	1	1	1	1	1	1	1	1

- Gamma-maximin: {right}
- Gamma-maximax: {left}

- interval maximal: {left, right}
- robust Bayes maximal: {left, right}

Risky Investment

	improve	*remain*	*worsen*	*p0*	*p1*	*lpr*	*upr*
improve				0	0.3		
remain				0.6	0.3		
worsen				0.4	0.4		
d1	100	50	-25	20	35	20	35
d2	75	50	0	30	37.5	30	37.5
d3	60	55	10	37	38.5	37	38.5
d4	35	35	35	35	35	35	35

- Gamma-maximin: {3}
- Gamma-maximax: {3}
- interval maximal: {2, 3}
- robust Bayes maximal: {3}

Solution to Exercise 6:

- c<37
    - Gamma-maximin: {3}
    - Gamma-maximax: {3}
    - interval maximal: {2, 3}
    - robust Bayes maximal: {3}
- 37<c<37.5
    - Gamma-maximin: {4}
    - Gamma-maximax: {3}
    - interval maximal: {2, 3, 4}
    - robust Bayes maximal: {3, 4}
- 37.5<c<38.5
    - Gamma-maximin: {4}
    - Gamma-maximax: {3}
    - interval maximal: {3, 4}
    - robust Bayes maximal: {3, 4}
- 38.5<c
    - Gamma-maximin: {4}
    - Gamma-maximax: {4}
    - interval maximal: {4}
    - robust Bayes maximal: {4}

Oil Wildcatter

Normal Form Backward Induction

T1 Branch

	S1	*S2*	*p0*	*p1*	*p2*	*lpr*	*upr*
S1			0.75	0.667	0.833		
S2			0.25	0.333	0.167		
d1	-1	-1	-1	-1	-1	-1	-1
d2	-6	10	-2	-0.672	-3.328	-3.328	-0.672

So {d1,d2} is robust Bayes maximal in the T1 branch.

T2 Branch

	S1	*S2*	*p0*	*p1*	*p2*	*lpr*	*upr*
S1			0.167	0.333	0.25		
S2			0.833	0.667	0.75		
d1	-1	-1	-1	-1	-1	-1	-1
d2	-6	10	7.328	4.672	6	4.672	7.328

So {d2} is robust Bayes maximal in the T2 branch.

dTc branch

	T1S1	*T1S2*	*T2S1*	*T2S2*	*p0*	*p1*	*p2*	*p3*	*lpr*	*upr*
T1S1					0.3	0.3	0.4	0.5		
T1S2					0.1	0.1	0.2	0.1		
T2S1					0.1	0.2	0.1	0.1		
T2S2					0.5	0.4	0.3	0.3		
d1	0	0	0	0	0	0	0	0	0	0
d2	-5	11	-5	11	4.6	3	3	1.4	1.4	4.6

So {d2} is robust Bayes maximal in the dTc branch.

dT Branch

	T1S1	*T1S2*	*T2S1*	*T2S2*	*p0*	*p1*	*p2*	*p3*	*lpr*	*upr*
T1S1					0.3	0.3	0.4	0.5		
T1S2					0.1	0.1	0.2	0.1		
T2S1					0.1	0.2	0.1	0.1		
T2S2					0.5	0.4	0.3	0.3		
s1	-1	-1	-6	10	4	2.4	1.8	1.8	1.8	4

s2	-6	10	-6	10	3.6	2	2	0.4	0.4	3.6

So both strategies {s1,s2} are robust Bayes maximal in the dT branch.

Root node:

	T1S1	*T1S2*	*T2S1*	*T2S2*	*p0*	*p1*	*p2*	*p3*	*lpr*	*upr*
T1S1					0.3	0.3	0.4	0.5		
T1S2					0.1	0.1	0.2	0.1		
T2S1					0.1	0.2	0.1	0.1		
T2S2					0.5	0.4	0.3	0.3		
s1	-1	-1	-6	10	4	2.4	1.8	1.8	1.8	4
s2	-6	10	-6	10	3.6	2	2	0.4	0.4	3.6
d2	-5	11	-5	11	4.6	3	3	1.4	1.4	4.6

So {s1,d2} is robust Bayes maximal.

Toy Example

Note, for brevity, the tables below write D1 for delta1 and D2 for delta2.

Normal Form

	S1E1	*S1E2*	*S2*	*p0*	*p1*	*lpr*	*upr*
S1E1				0.2	0.1		
S1E2				0.3	0.4		
S2				0.5	0.5		
d1(S1 D1)	1	4	2	2.4	2.7	2.4	2.7
d1(S1 D2)	2.5	2.5	2	2.25	2.25	2.25	2.25
d2	2.3	2.3	2.3	2.3	2.3	2.3	2.3

So {d1(S1 delta1)} is robust Bayes maximal.

Normal Form Backward Induction

S1 Branch

	E1	*E2*	*p0*	*p1*	*lpr*	*upr*
E1			0.4	0.2		
E2			0.6	0.8		
D1	1	4	2.8	3.4	2.8	3.4
D2	2.5	2.5	2.5	2.5	2.5	2.5

So {delta1} is robust Bayes maximal.

Root Node

	S1E1	*S1E2*	*S2*	*p0*	*p1*	*lpr*	*upr*
S1E1				0.2	0.1		
S1E2				0.3	0.4		
S2				0.5	0.5		
d1(S1 D1)	1	4	2	2.4	2.7	2.4	2.7
d2	2.3	2.3	2.3	2.3	2.3	2.3	2.3

So {d1(S1 delta1)} is robust Bayes maximal.

Extra Question

d2 becomes uniquely robust Bayes maximal if it is assigned a value strictly larger than 2.7.

Lake District

Normal Form Backward Induction

S1 Branch

	E1	*E2*	*p0*	*p1*	*p2*	*lpr*	*upr*
E1			0.591	0.7	0.747		
E2			0.409	0.3	0.253		
d1	9	14	11.045	10.5	10.265	10.265	11.045
d2	4	19	10.135	8.5	7.795	7.795	10.135

So {d1} is robust Bayes maximal in the S1 branch.

S2 Branch

	E1	*E2*	*p0*	*p1*	*p2*	*lpr*	*upr*
E1			0.157	0.2	0.374		
E2			0.843	0.8	0.626		
d1	9	14	13.215	13	12.13	12.13	13.215
d2	4	19	16.645	16	13.39	13.39	16.645

So {d2} is robust Bayes maximal in the S1 branch.

dSc Branch

	S1E1	*S1E2*	*S2E1*	*S2E2*	*p0*	*p1*	*p2*	*p3*	*lpr*	*upr*
S1E1					0.378	0.378	0.378	0.478		
S1E2					0.162	0.162	0.262	0.162		
S2E1					0.072	0.172	0.072	0.072		
S2E2					0.388	0.288	0.288	0.288		
d1	10	15	10	15	12.75	12.25	12.75	12.25	12.25	12.75

d2	5	20	5	20	13.25	11.75	13.25	11.75	11.75	13.25

So both options {d1,d2} are robust Bayes maximal in the dSc branch.

dS Branch

The solution is trivial, as there is only a single gamble: 9, 14, 4, 19.

Root node

	S1E1	*S1E2*	*S2E1*	*S2E2*	*p0*	*p1*	*p2*	*p3*	*lpr*	*upr*
S1E1					0.378	0.378	0.378	0.478		
S1E2					0.162	0.162	0.262	0.162		
S2E1					0.072	0.172	0.072	0.072		
S2E2					0.388	0.288	0.288	0.288		
s1	9	14	4	19	13.33	11.83	12.83	12.33	11.83	13.33
d1	10	15	10	15	12.75	12.25	12.75	12.25	12.25	12.75
d2	5	20	5	20	13.25	11.75	13.25	11.75	11.75	13.25

So all three remaining normal form decisions {s1,d1,d2} are robust Bayes maximal.

Value of Information

A simple check reveals that s1 is no longer robust Bayes maximal as soon as c exceeds 1.58 (this would increase the lower prevision of d1 - s1 by 0.58, which is just enough to make it zero).